6.1. *Construction from a sidelength and the corresponding median and angle bisector.* Given the length $2a$ of a side of a triangle, and the lengths m and t of the median and the angle bisector on the same side, to construct the triangle. This is Problem 1054(a) of the *Mathematics Magazine* [6]. In his solution, Howard Eves denotes by z the distance between the midpoint and the foot of the angle bisector on the side $2a$, and obtains the equation

$$z^4 - (m^2 + t^2 + a^2)z^2 + a^2(m^2 - t^2) = 0,$$

from which he concludes constructibility (by ruler and compass). We devise a simple construction, assuming the data given in the form of a triangle $AM'T$ with $AT = t$, $AM' = m$ and $M'T = a$. See Figure 33. Writing $a^2 = m^2 + t^2 - 2tu$, and $z^2 = m^2 + t^2 - 2tw$, we simplify the above equation into

$$w(w-u) = \frac{1}{2}a^2. \tag{4}$$

Note that u is length of the projection of AM' on the line AT, and w is the length of the median AM on the bisector AT of the sought triangle ABC. The length w can be easily constructed, from this it is easy to complete the triangle ABC.

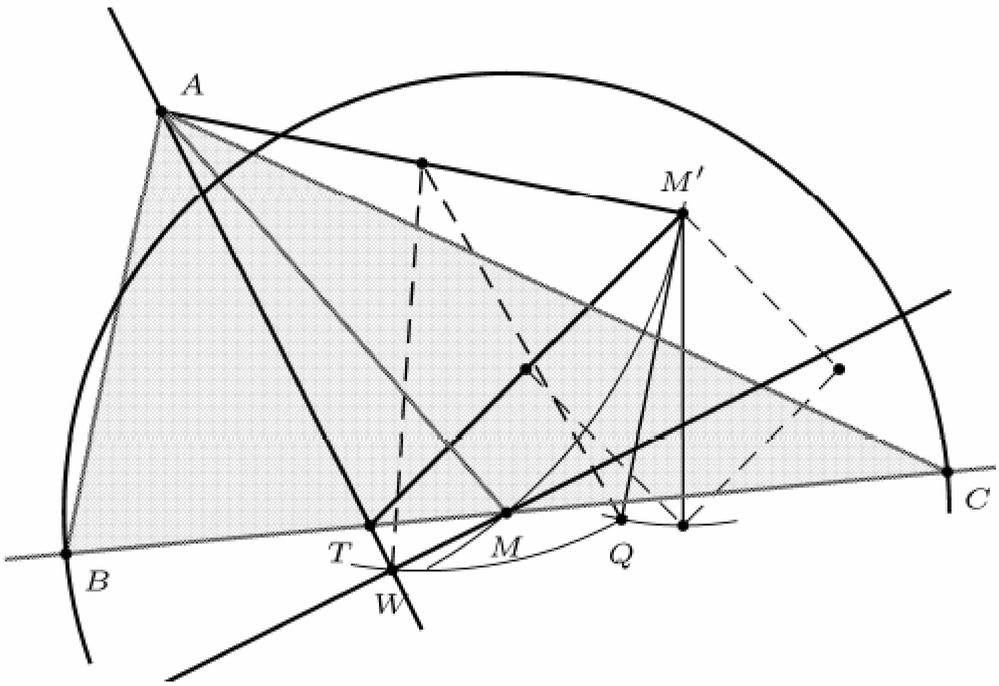

Figure 33