

Propuesto por Jean Louis Aymé

Problema 845

1. Sea ABCD un cuadrilátero convexo
2.  $E_u$  el punto Euler-Poncelet de ABCD
3. PQR el triángulo diagonal de ABCD
4. (O) el circuncírculo de PQR.

Demostrar que (O) contiene a  $E_u$ .

Demostración usando coordenadas baricéntricas

Angel Montesdeoca

**Euler-Poncelet Point** in ENCYCLOPEDIA OF QUADRI-FIGURES (EQF)  
<http://chrisvantienhoven.nl/mathematics/encyclopedia> (Chris van Tienhoven)

Let  $P_1, P_2, P_3, P_4$  be the defining Quadrangle Points.

Let  $S_1 = P_1.P_2 \wedge P_3.P_4$ ,  $S_2 = P_1.P_3 \wedge P_2.P_4$  and  $S_3 = P_1.P_4 \wedge P_2.P_3$ .

Now  $S_1.S_2.S_3$  is the QA-Diagonal Triangle of the Reference Quadrangle.

( <http://www.chrisvantienhoven.nl/other-quadrangle-objects/15-mathematics/quadrangle-objects/artikelen-qa/90-qa-tr1.html> )

QA-DT-Coordinate system, where the QA-Diagonal Triangle is defined as the Reference Triangle with barycentric vertex coordinates  $(1:0:0)$ ,  $(0:1:0)$ ,  $(0:0:1)$ . An arbitrary point of the Quadrangle is defined as  $(p:q:r)$ . The other 3 points now form the Anticevian triangle of  $P_i$  wrt the QA-Diagonal Triangle and have vertices  $(-p : q : r)$ ,  $(p : -q : r)$ ,  $(p : q : -r)$ .

(<http://www.chrisvantienhoven.nl/quadrangle-objects/15-mathematics/quadrangle-objects/artikelen-qa/31-qa-1.html>)

Euler-Poncelet Point

( <http://www.chrisvantienhoven.nl/quadrangle-objects/10-mathematics/quadrangle-objects/12-qa-p2.html> )

Euler mentioned this point in one of his numerous papers.

In 1821 Brianchon and Poncelet, both captains of artillery, wrote a book (Ref-1) in which the orthogonal hyperbola and this point were worked out.

The Euler-Poncelet Point can be defined in different ways.

1. It is the center of the orthogonal hyperbola through  $P_1, P_2, P_3$  and  $P_4$ .
2. It is the common point of the Nine-point Circles of the triangles  $P_j.P_k.P_l$  for all combinations of  $(j,k,l) \in (1,2,3,4)$ .
3. It is the common point of the Pedal Circles of points  $P_i$  wrt triangles  $P_j.P_k.P_l$  for all combinations of  $(i,j,k,l) \in (1,2,3,4)$ .

This point is also described at Ref-2c, Ref-15f and Ref-42.

Coordinates:

1st DT-Coordinate:

$$1/(b^2 r^2 - c^2 q^2)$$

Properties:

QA-P2 lies on the circumcircle (  $a^2 y z + b^2 x z + c^2 x y = 0$  ) of the QA-Diagonal Triangle.

[1] Brianchon C. J., Poncelet J.-V., Recherche sur la détermination d'une hyperbole équilatère au moyende quatre conditions données, Annales Mathématiques de 1821

\* Page 512 about Euler Circles.

[2c] Jean-Louis Ayme, Le Point- d'Euler-Poncelet d'un Quadrilatère, available at <http://perso.orange.fr/jl.ayme> vol. 8 Le point d'Euler-Poncelet d'un quadrilatère.

\* Page 11 Synthetical proof midcircles are concurrent

[15f] 11-3 Miquel-, Poncelet- und Bennett-Punkt eines Vierecks:

<http://eckartschmidt.de/Pktve.pdf> (QL-P1/-P2/-P4)

[42] Darij Grinberg, Poncelet points and antigonal conjugates,

available at: <http://www.mathlinks.ro/Forum/viewtopic.php?t=109112>.