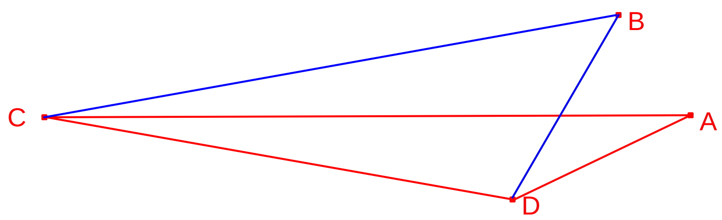
Problema 846

Siga un triangle $\overset{\Delta}{ACD}$. Siga un punt B exterior al triangle.

$\angle ACB = 10^{o}$, $\angle BAC = 55^{o}$, $\perp CBD = 50^{o}$, $\angle CAD = 25^{o}$.

Determineu la mesura de l'angle $\angle ACD$.



Solució de Ricard Peiró i Estruch:

Siga $x = \angle ACD$.

Aplicant el teorema dels sinus al triangle $\overset{\Delta}{BCD}$:

$$\frac{\overline{BD}}{\sin(10^{o}+x)} = \frac{\overline{CD}}{\sin 50^{o}} \qquad (1)$$

Aplicant el teorema dels sinus al triangle $\overset{\Delta}{ABD}$:

$$\frac{\overline{BD}}{\sin 80^{o}} = \frac{\overline{AB}}{\sin 35^{o}} \qquad (2)$$

Dividint les expressions (1) (2):

$$\frac{\overline{CD}}{\overline{AB}} = \frac{\sin 80^{o} \cdot \sin 50^{o}}{\sin(10+x) \cdot \sin 35^{o}} \qquad (3)$$

Aplicant el teorema dels sinus al triangle $\overset{\Delta}{ACD}$:

$$\frac{\overline{AD}}{\sin x} = \frac{\overline{CD}}{\sin 25^{o}} \qquad (4)$$

Aplicant el teorema dels sinus al triangle $\overset{\Delta}{ABD}$:

$$\frac{\overline{AD}}{\sin 65^{o}} = \frac{\overline{AB}}{\sin 35^{o}} \qquad (5)$$

Dividint les expressions (4) (5):

$$\frac{\overline{CD}}{\overline{AB}} = \frac{\sin 65^{o} \cdot \sin 25^{o}}{\sin x \cdot \sin 35^{o}} \qquad (6)$$

Igualant les expressions (3) (6):

$$\frac{\sin 80^{o} \cdot \sin 50^{o}}{\sin(10^{o}+x) \cdot \sin 35^{o}} = \frac{\sin 65^{o} \cdot \sin 25^{o}}{\sin x \cdot \sin 35^{o}} \qquad (7)$$

Simplificant:

$$\frac{\cos 10^{o} \cdot \cos 40^{o}}{\sin(10^{o}+x)} = \frac{\frac{1}{2}\cos 40^{o}}{\sin x} \qquad (8)$$

Simplificant:

$$\sin(10^{o}+x) = 2\sin x \cdot \cos 10^{o} \qquad (9)$$

$$\sin(10^{o}+x) = \sin(x-10^{o}) + \sin(10^{o}+x) \qquad 10)$$

Simplificant :

$\sin(x - 10^{o}) = 0$.

Aleshores, $x = 10^{o}$