

13. (A.Myakishev, 9–10) Given triangle ABC . One of its excircles is tangent to the side BC at point A_1 and to the extensions of two other sides. Another excircle is tangent to side AC at point B_1 . Segments AA_1 and BB_1 meet at point N . Point P is chosen on the ray AA_1 so that $AP = NA_1$. Prove that P lies on the incircle.

Solution. Since the points of contact of the sides of the triangle with the excircles are symmetric to the points of contact with the incircle in the midpoints of sides, $CA_1 = p - b$, $CB_1 = p - a$, $AB_1 = BA_1 = p - c$. Menelaus theorem applied to triangle ACA_1 and line BB_1 implies $A_1N/AA_1 = (p - a)/p$. Homothety with this factor and center A transfers point A_1 to point P . But the ratio of the radii of the incircle and the excircle of the triangle also equals $(p - a)/p$, thus the image of A_1 under the homothety lies on the incircle (fig.13).

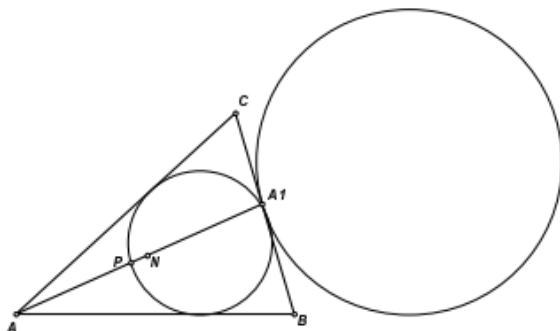


Fig. 13