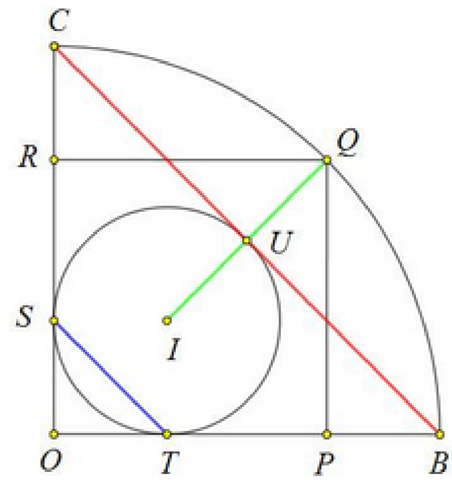
- ⋆ $\odot(OBC)$ quarter of circle (O, OB)
- ⋆ I incenter of $\triangle OBC$
- ⋆ S, T, U touch points
- ⋆ $OPBR$ square inscribed in mixtilinear triangle OBC



Prove that: $\dfrac{1}{BC} + \dfrac{1}{IQ} = \dfrac{1}{TS}$

Propuesto por Ecole Suppa
Solución
Supongamos que $OP = OR = RQ = QP = a$.Entonces

$$OQ = OB = OC = a\sqrt{2}$$

y

$$CB = 2a$$

El radio,r_1 del íncírculo del $\triangle OCB$ es

$$r_1 = \frac{S_{\triangle OCB}}{semiper\acute{\imath}metro\triangle OCB} = \frac{a^2}{a(\sqrt{2}+1)} = a\left(\sqrt{2}-1\right)$$

$IT = IS = r_1 = a\left(\sqrt{2}-1\right)$
Fíjémonos que

$$\begin{bmatrix} TS = OI = \sqrt{2}IT = a\sqrt{2}\left(\sqrt{2}-1\right) = a\left(2-\sqrt{2}\right) \\ IQ = OQ - OI = a\sqrt{2} - a\left(2-\sqrt{2}\right) = 2\left(\sqrt{2}-1\right)a \end{bmatrix}$$

- Vamos ahora calcular $\frac{1}{BC} + \frac{1}{IQ}$

$$\frac{1}{BC} + \frac{1}{IQ} = \frac{1}{2a} + \frac{1}{2\left(\sqrt{2}-1\right)a} = \frac{\sqrt{2}}{2\left(\sqrt{2}-1\right)a} = \frac{1}{\sqrt{2}\left(\sqrt{2}-1\right)a} = \frac{1}{(2-\sqrt{2})a} \tag{1}$$

- Como $\frac{1}{TS}$

$$\frac{1}{TS} = \frac{1}{a\left(2-\sqrt{2}\right)} \tag{2}$$

De (1) y (2) concluimos que

$$\frac{1}{BC} + \frac{1}{IQ} = \frac{1}{TS}$$