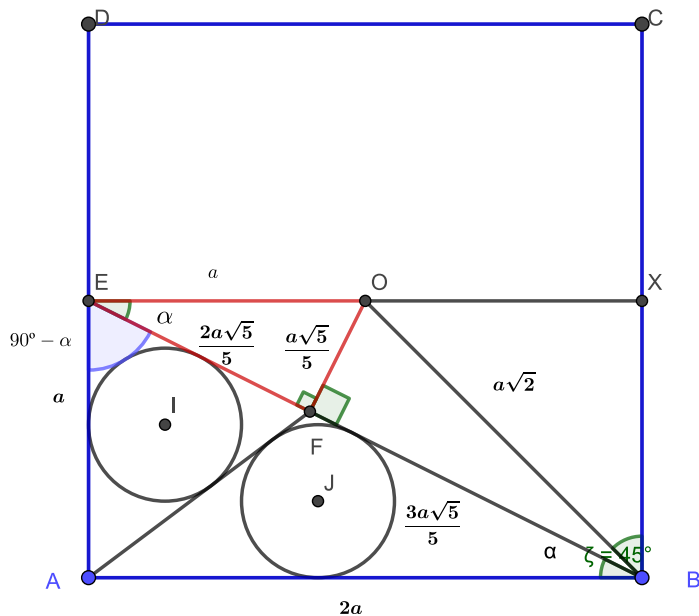




Por el teorema del coseno

$$AF^2 = AE^2 + EF^2 - 2AE \cdot EF \cos(90^\circ - \alpha)$$

$$AF^2 = a^2 + \frac{4a^2}{5} - \frac{4a^2\sqrt{5}}{5} \cdot \frac{\sqrt{5}}{5} = a^2$$

$$AF = a$$

$$\text{Semiperímetro } \Delta_{AEF} = \frac{1}{2} (2a + \frac{2a\sqrt{5}}{5}) = (a + \frac{a\sqrt{5}}{5})$$

$$\text{Superficie } \Delta_{AEF} = \frac{1}{2} EA \cdot EF \sin(90^\circ - \alpha) = \frac{2a^2}{5}$$

$$\text{radio}(I) = \frac{\text{Superficie } \Delta_{AEF}}{\text{Semiperímetro } \Delta_{AEF}} = \frac{2a}{5 + \sqrt{5}}$$

$$AB = 2a$$

$$AE = a = OX$$

$$EB = a\sqrt{5} \quad OB = a\sqrt{2}$$

$$\Delta AEB \sim \Delta DFE$$

$$\text{Como } \frac{EO}{EB} = \frac{1}{\sqrt{5}} \quad \text{entonces}$$

$$EF = \frac{AB}{\sqrt{5}} = \frac{2a}{\sqrt{5}} = \frac{2a\sqrt{5}}{5}$$

$$OF = \frac{AE}{\sqrt{5}} = \frac{a}{\sqrt{5}} = \frac{a\sqrt{5}}{5}$$

$$FB = \sqrt{OB^2 - OF^2} = \frac{3a}{\sqrt{5}} = \frac{3a\sqrt{5}}{5}$$

$$\tan \alpha = \frac{1}{2}$$

$$\cos^2 \alpha = \frac{1}{1 + \tan^2 \alpha} = \frac{1}{1 + \frac{1}{4}} = \frac{4}{5} \Leftrightarrow \cos \alpha = \frac{2\sqrt{5}}{5}$$

$$\sin^2 \alpha = 1 - \cos^2 \alpha = \frac{1}{5} \Leftrightarrow \sin \alpha = \frac{\sqrt{5}}{5}$$

$$\text{Semiperímetro } \Delta_{BFA} = \frac{1}{2} (3a + \frac{3a\sqrt{5}}{5}) = \frac{3}{2} (a + \frac{a\sqrt{5}}{5})$$

$$\text{Superficie } \Delta_{BFA} = \text{Superficie } \Delta_{ABE} - \text{Superficie } \Delta_{AEF} = a^2 - \frac{2a^2}{5} = \frac{3a^2}{5}$$

$$\text{radio}(J) = \frac{\text{Superficie } \Delta_{BFA}}{\text{Semiperímetro } \Delta_{BFA}} = \frac{2a}{5 + \sqrt{5}}$$