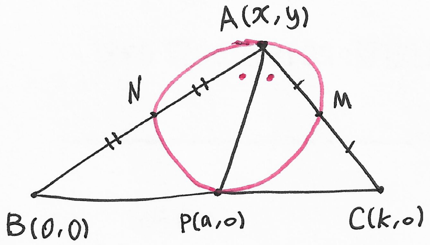
Triangulos Cabri Problem 979 HyunBin Yoo, South Korea



Let $B(0,0)$, $C(k,0)$, $A(x,y)$ and the tangent $P(a,0)$.

Then $BP = a$ and $CP = k-a$.

Since $BP^2 = BN \cdot BA$, $BA = \sqrt{2}a$. Also $CA = \sqrt{2}(k-a)$.

Since $AB : AC = BP : CP$, AP is the angle bisector of BAC.

Using the distance of AB and AC, the following system of equations can be made :

$$\begin{cases} x^2 + y^2 = 2a^2 \\ (x-k)^2 + y^2 = 2(k-a)^2 \end{cases}$$

Solving in terms of x leads to $x = 2a - \frac{k}{2}$. So $a = \frac{x}{2} + \frac{k}{4}$.

Substitute a with $\frac{x}{2} + \frac{k}{4}$ to get an equation without a.

$$x^2 + y^2 = 2\left(\frac{x}{2} + \frac{k}{4}\right)^2 \iff x^2 + y^2 = \frac{x^2}{2} + \frac{kx}{2} + \frac{k^2}{8}$$

$$\iff x^2 - kx + 2y^2 = \frac{k^2}{4}$$

$$\iff \left(x - \frac{k}{2}\right)^2 + 2y^2 = \frac{k^2}{2}$$

Thus the locus of A is an ellipse whose foci are $\underset{B}{(0,0)}$ and $\underset{C}{(k,0)}$, excluding its vertices $\left(\frac{1 \pm \sqrt{2}}{2}k, 0\right)$.