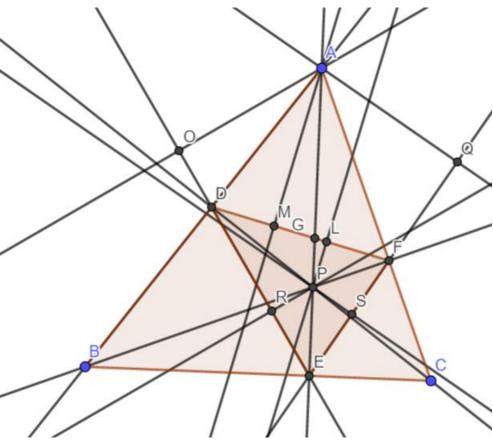
12190. Proposed by Leonard Giugiuc, Drobeta Turnu Severin, Romania, and Gabriela Negutescu, Telea, Romania. Let ABC be a triangle, and let D, E, and F be points on AB, BC, and CA, respectively, such that AD, BE, and CF are concurrent at P. It is well known that if P is the orthocenter of ABC, then P is the incenter of DEF. Prove the converse.

Solution of Ricardo Barroso Campos. Sevilla. Spain.

First it must be AE, BF and CD concurrent at P.



If P is the incenter of DEF, let G =AE \cap DF Let O, M, and Q projection of A on ED, DF, and EF. Let R, L and S projection of P on ED, DF, and EF.

Is:
$$\frac{AO}{PR} = \frac{AM}{PI} = \frac{AQ}{PS} = \frac{AG}{PG}$$

Is: $\frac{AO}{PR}=\frac{AM}{PL}=\frac{AQ}{PS}=\frac{AG}{PG}$ Thus A is the exincenter of DEF triangle.

For analogous reasoning, B and C are exincenters of DEF triangle. Thus, $AE \perp BC$, $BF \perp AC$, and $CD \perp BA$, and P is the orthocenter of ABC.